(NASA-CR-170750) RELATIONS BETWEEN MIRROR FLATS GEOMETRY AND X-RAY DATA (Alabama A & M Univ., Normal.) 13 p HC A02/MF A01 CSCL 20F

N83-25544

Unclas
G3/74 11659

TECHNICAL REPORT NO. 2

RELATIONS BETWEEN MIRROR FLATS GEOMETRY AND X-RAY DATA

Prepared for

NASA/MSFC
George C. Marshall Space Flight Center
Huntsville, AL 35812

Contract NAS8-34591

By

Alton C. Williams
Alabama A. & M. University
Normal, AL 35762

January 1982

MIRROR FLATS GEOMETRY

I. INTRODUCTION

Presently, the x-ray data itself is being used to determine the position where the mirror flats are properly aligned. That is, the zero angle of incidence position of the mirrors is determined with the data. The mirrors are in the periscope geometry as shown in Fig. 1. However, the value of S_T, the distance from the slit to the plane of the short mirror, is not fixed nor known a priori. This parameter plays a very important role in the zero position of the mirrors.

As a further complicating factor, it is possible that the mirrors are not parallel. That is, they may be "cocked" by some angles α and β, respectively, as shown in Fig. 2.

In the following, the relation between the x-ray data and the geometry of the mirrors is given. Further, the experimental errors are given in terms of known or measured parameters.

II. DEFINITION OF PARAMETERS

Fig. 1 is a diagram of the "uncocked" mirrors arrangement. The parameters that will be used in this discussion are indicated on the diagram. These parameters are the following: (The numerical values in parentheses are the nominal or fixed values)

d(.012 in.) -- The width of the slit;

w(.03125 in.) -- The distance between the mirror surfaces;

S_T -- The distance from the top of the slit to the plane of the short mirror;

L_s(4.0 in.) -- The length of the short mirror;

L_L(6.5 in.) -- The length of the long mirror;

L_1(4.173 in.) -- The distance from the plane of the slit to the far end of the short mirror;

L_2(8.533 in.) -- The distance from the plane of the slit to the far end of the long mirror;

δ_1(.005 -- .05 in) -- The uncoated width of the near end of the short mirror;

δ_2(.005 -- .05 in) -- The uncoated width of the far end of the short mirror;

Δ_1(.005 -- .05 in) -- The uncoated width of the near end of the long mirror;

Δ_2(.005 -- .05 in) -- The uncoated width of the far end of the long mirror.

Fig. 2. Illustrates the "cock" of the Mirrors.

It is assumed that the mirrors are cocked by a rotation about an axis which is perpendicular to the plane of the diagram and passes through the center of the overlap region of the two mirrors. The center of the overlap region is a distance D from the plane of the slit, and it is given by

$$D = \tfrac{1}{2}(L_1 + L_2 - L_L) \qquad (1)$$

The angle of cock of the short mirror is denoted by α. A clockwise rotation of the short mirror corresponds to a positive α. The angle of cock of the long mirror is denoted by β. A clockwise rotation of the long mirror corresponds to a negative β. Note that $\alpha + \beta$ could be zero which would indicate that the mirrors are parallel, but the mirrors could still cocked overall. That is $\alpha = -\beta \neq 0$.

III. ANGLE WHERE DIRECT BEAM FIRST APPEARS

If the mirrors are rotated counterclockwise until the entire beam is being completely reflected by the long mirror, no x-rays will be seen at the detector because they will be blocked by the boundary of the tube before they reach the plane of the detector. The angle at which this first occurs is denoted by θ_1 and it is related to the geometrical parameters of the mirrors via the relation

$$\theta_1 = \frac{S_T - W + (L_2 - D)\beta}{L_2} \tag{2}$$

IV. ANGLE WHERE DIRECT BEAM REACHES ITS MAXIMUM

(Angle Where Beam Begins To Be Blocked By Long Mirror)

When the mirrors are rotated counterclockwise so that the x-rays are no longer detected, one can now rotate the mirrors clockwise until the direct beam is again detected. The angle at which the direct beam first reaches its maximum is where the x-rays are no longer being reflected by the long mirror. This angle is denoted by θ_2 and is given by the relation

$$\theta_2 = \theta_1 + \frac{d}{L_2} \tag{3}$$

Where θ_1 is given in Eq. (2).

V. ANGLE WHERE DIRECT BEAM BEGINS TO BE BLOCKED BY SHORT MIRROR

If the mirrors are further rotated clockwise, the beam will begin to be reflected from the short mirror. This single reflection will lead to a reduction in the amount of direct beam which reaches the detector. The angle at which this first begins is denoted by θ_3 and is given by

$$\theta_3 = \frac{S_T - (L_1 - D)\alpha}{L_1} \tag{4}$$

VI. ANGLE WHERE ENTIRE BEAM IS BEING REFLECTED FROM SHORT MIRROR

As the mirrors are further rotated clockwise, eventually all of the direct beam will begin to be reflected from the short mirror. The angle at which this occurs is denoted by θ_4 and it is given by

$$\theta_4 = \theta_3 + \frac{d}{L_1} \tag{5}$$

VII. ANGLE WHERE DOUBLE REFLECTION FIRST BEGIN

At some point, the x-rays reflected from the short mirror will be reflected again by the long mirror. When this occurs, the reflected x-rays will be detected at the plane of the detector because of the periscope geometry of the mirrors. The angle at which this occursis denoted by θ_5 and it is given by

$$\theta_5 = \frac{S_T + W + D(\alpha+\beta) - L_2(2\alpha+\beta) + \left\{\left[S_T + W + D(\alpha+\beta) - L_2(2\alpha+\beta)\right]^2 + 4\alpha S_T L_2\right\}^{\frac{1}{2}}}{2L_2} \qquad (6)$$

VIII. CALCULATED VALUE OF S_T

If the angular differences $\theta_5 - \theta_4$ and $\theta_4 - \theta_1$ are measured, the distance from the top of the slit to the plane of the short mirror can be calculated with the equation:

$$S_T = \frac{H^2 - G^2}{2FG - 2QH - R} \qquad (7)$$

Where

$$H = W + DP - L_2(2P - A) \qquad (8)$$

$$G = 2L_2\left[\theta_5 - \theta_4 + \frac{d}{L_1} - \frac{(L_1 - D)(2P - A)}{L_1} - \frac{(W + DP)}{2L_2} + P - \frac{A}{2}\right] \qquad (9)$$

$$Q = 1 + L_2/D \qquad (10)$$

$$R = 4(P - A)\,L_2 \qquad (11)$$

$$F = \frac{L_2}{D} - 1 \tag{12}$$

$$A = \frac{L_1 L_2}{(L_2 - L_1)D} \left[\frac{d}{L_1} + \frac{W}{L_2} + P\left(\frac{D}{L_2} - 1\right) - (\theta_4 - \theta_1) \right] \tag{13}$$

$$P = \alpha + \beta \quad \text{(measured optically)} \tag{14}$$

IX. CALCULATED VALUES FOR α AND β

Once S_T is calculated with Eq. (7), the angles that the two mirrors are cocked can be determined from the equations below:

$$\beta = A + S_T/D \tag{15}$$

$$\alpha = P - \beta \tag{16}$$

Where A and P are given in Eqs. (13) and (14), respectively.

X. CALCULATION OF ZERO POSITION (VOLTAGE)

After the values of S_T, α, and β are determined, the zero position voltage can be calculated using any one of the measured angles. For example, if the angle θ_4 of Eq. (5) is used, one can write the Eq.

$$V_{\theta_4} = \theta_3 C + \left(\frac{d}{L_1}\right) C + V_{zero} \tag{17}$$

Where V_{θ_4} is the voltage reading which corresponds to the angle θ_4. C is a conversion factor which converts from an angular change in radians to an angular change in mV. Presently the equipment is arranged so that 19.2 mV corresponds to 1 arcmin. Using Eq. (4) for θ_3, Eq. (17) becomes

$$V_{\theta_4} = \frac{\left[S_T + d - (L_1 - D)\alpha\right]C}{L_1} + V_{zero} \qquad (18)$$

So that the zero voltage position is

$$V_{zero} = V_{\theta_4} - \frac{\left[S_T + d - (L_1 - D)\alpha\right]C}{L_1} \; . \qquad (19)$$

XI. ERROR ANALYSIS

If the quantities in the above discussion are calculated as indicated, then the errors (standard deviations) in the values of S_T, α, β, and V_{zero} are as follows

$$(\delta S_T)^2 = \frac{4 S_T^2 (H + QS_T)^2}{(H^2 - G^2)^2} \left[(D - 2 L_2)^2 (\delta P)^2 + L_2^2 (\delta A)^2\right] + \frac{4 S_T^2 (G + FS_T)^2}{(H^2 - G^2)^2} (\delta G)^2 + \frac{16 S_T^4 L_2^2}{(H^2 - G^2)^2}\left[(\delta P)^2 + (\delta A)^2\right] \qquad (20)$$

$$(\delta \alpha)^2 = (\delta P)^2 + (\delta A)^2 + \frac{(\delta S_T)^2}{D^2} \qquad (21)$$

$$(\delta\beta)^2 = (\delta A)^2 + \frac{(\delta S_T)^2}{D^2} \qquad (22)$$

$$(\delta V_{zero})^2 = (\delta V_\theta)^2 + C^2 \frac{(\delta S_T)^2}{L_1^{\,2}} + \frac{C^2(L_1 - D)^2}{L_1^{\,2}} (\delta\alpha)^2 \qquad (23)$$

Where δP is the uncertainty in the optical measurement of the overall cock angle $(\alpha + \beta)$; δV_θ is the accuracy to which the various angles are determined in the x-ray measurements; and the other quantities are defined by

$$(\delta A)^2 = \frac{L_1^{\,2} L_2^{\,2}}{(L_2 - L_1)^2 D^2} \left[\left(\frac{D}{L_1} - 1\right)^2 (\delta P)^2 + \frac{2(\delta V_\theta)^2}{C^2} \right] \qquad (24)$$

$$(\delta G)^2 = \frac{8L_2^{\,2}}{C^2} (\delta V_\theta)^2 + \left[2 L_2 - D - \frac{4 L_2 (L_1 - D)}{L_1}\right]^2 (\delta P)^2$$

$$+ \left[\frac{2 L_2 (L_1 - D)}{L_1} - L_2\right]^2 (\delta A)^2 \qquad (25)$$

XII. ACCEPTABLE RANGE FOR S_T

To ensure that all of the x-ray beam between the grazing angles of incidence of 25' to 51' hits the coated part of the two mirrors, there is a restricted range of values for the parameter S_T (see Fig. 1). Neglecting the possible "cock" in the mirrors which should not affect this range appreciably, the acceptable range of S_T is the following:

$$\text{The maximum of} \begin{cases} (L_2 - L_L + D_1) \text{ Tan } 51' - W < S_T < \\ (L_1 - L_s + \delta_1) \text{ Tan } 51' \end{cases}$$

$$\text{The minimum of} \begin{cases} (L_2 - \Delta_2) \text{ Tan } 25' - W - d \\ (L_1 - \delta_2) \text{ Tan } 25' - d \end{cases} \tag{26}$$

Typically, this requires that

$$.003 \text{ in} < S_T < .017 \text{ in} \tag{27}$$

XIII. A TYPICAL EXAMPLE

As a typical example, the mirror EK-IV-2 (Ceruit/Pt) mirror will be used. Using the HRI detector it was determined that

V_{θ_1} = 610 mV

V_{θ_3} = 910 mV

V_{θ_4} = 1120 mV

V_{θ_5} = 1090 mV

These values were measured to an accuracy of $\pm$ 2 mV.

The overall "cock" angle of the mirrors was determined from optical measurements to be 13". The accuracy in this value is assumed to be $\pm$ 2". That is

$$\delta P = 2'' = 9.696 \times 10^{-6} \text{ radian}$$

The parameters for this mirror are

$L_1 = 4.176$ in $\qquad \delta_1 = \delta_2 = \Delta_1 = \Delta_2 = .005$ in

$L_s = 4.000$ in $\qquad d = .012$ in

$L_2 = 8.533$ in $\qquad w = .03125$ in

$L_L = 6.498$ in

The conversion factor C is

$$C = 6.6 \times 10^4 \text{ mV/rad} .$$

Hence, one obtains for example:

$$\theta_5 - \theta_4 = \frac{V_{\theta_5} - V_{\theta_4}}{C} \doteq -4.55 \times 10^{-4} \text{ rad.}$$

Using Eq. 7 the value of S_T is

$$S_T = .010 \text{ in}$$

Eqs. (15) and (16) then give

$$\alpha = -20.8 \text{ arcsec}$$

$$\beta = 33.8 \text{ arcsec}$$

Eqs. (19) gives the zero voltage position as

$$V_{zero} = 765 \text{ mV}$$

The uncertainties as calculated by Eqs. (20) - (23) are

$$\delta S_T = .002 \text{ in}$$

$$\delta\alpha = \delta\beta = 148.6 \text{ arcsec}$$

$$\delta V_{zero} = 37 \text{ mV}$$

XIV. DISCUSSION OF TYPICAL EXAMPLE

The uncertainty in the measurement of V_θ is the dominant factor in the accuracy of the calculated parameters. The smallest practical uncertainty in V_θ is $\pm$ 2 mV which is what was used in the typical example of the last section. This gave an uncertainty in the value of S_T of $\pm$.002 in. This means that the acceptable range for the calculated value of S_T is now (see Eq. 27)

$$.005 \text{ in} < S_T < .015 \text{ in}$$

This uncertainty in V_θ gives an uncertainty in the zero position to be $\pm$ 37 mV which corresponds to a $\pm$ 2 arcmin uncertainty in the grazing angle of incidence.

This uncertainty in V_θ also gives an uncertainty in the angle of "cock" of the mirrors of over two arcminutes whereas the "cock" angles are of the order of a half of an arcminute. This means that the angles of "cock" of the mirrors cannot be determined with any reasonable degree of accuracy. On the other hand, with such a large uncertainty in the grazing angle of incidence, the small "cock" angles are insignificant.

FIG. 1 -- DEFINITION OF MIRRORS PARAMETERS

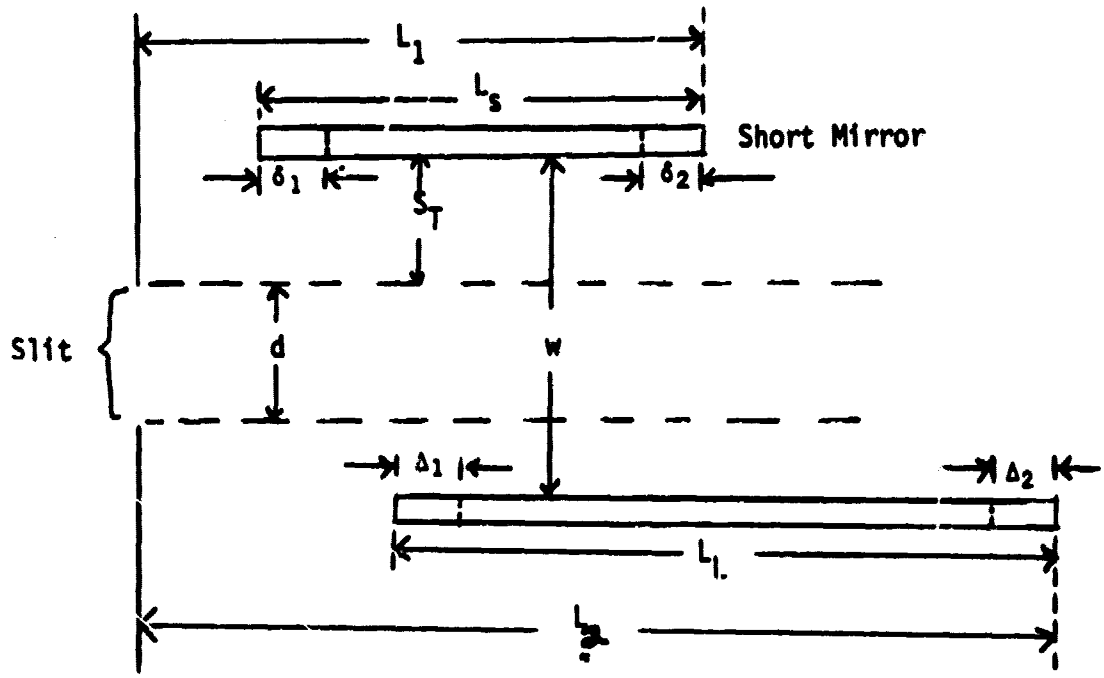

FIG. 2 -- DEFINITION OF "COCK" ANGLES OF MIRRORS